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John Roberts

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ABSTRACT

Amplitudes and travel times have been calculated for the first half-cycle of seismic P waves generated by nuclear explosions at distances varying from 140 to 2900 km. The head-wave-body-wave model constructed yields travel times within ±3.9 sec of the mean experimental values over the indicated range of distances and also provides an interpretation of the large-small-large nature of the amplitudes.

The work reported herein represents a partial interpretation of the experimental data displayed in Fig. 1 and of the type of data displayed in Figs. 2 and 3. Figure 1 shows the Jeffreys-Bullen travel time versus distance curve for seismic P waves.[1,2] Figure 2 shows the AFTAC-Gutenberg curve for the relative amplitude of P waves at 1 cps versus distance,[3,4] and Fig. 3 shows the amplitude at 1 cps versus distance for P waves from the Bilby and Wagtail tests.[3] The scatter of the points in Fig. 3 should be borne in mind when looking at Fig. 2. The curve in Fig. 2 represents a kind of median line through a broad smear. One should also bear in mind that the data presented in Figs. 2 and 3 represent measurements made a few cycles after the first and are probably not more than qualitatively comparable to the amplitude of the first half-cycle.

Considerable attention has been given to the curves of Figs. 1 and 2 in the past, although the emphasis has tended to be more on one or the other rather than on both simultaneously. British seismologists, led by Jeffreys, have devoted most of their effort to interpreting the travel time curve with particular attention to its marked change of slope in the vicinity of 2000 km. The change of slope was explained by assuming (1) that all first arrivals were body waves that passed through the mantle, (2) that the wave velocity increased continuously with depth in the mantle, and (3) that the increase was especially rapid over a certain region.[5] Gutenberg,[6] on the other hand, assumed the existence of a low-velocity layer in the mantle to explain the low amplitudes around 1000 km. The low velocity layer was envisaged as bending the rays incident upon it away

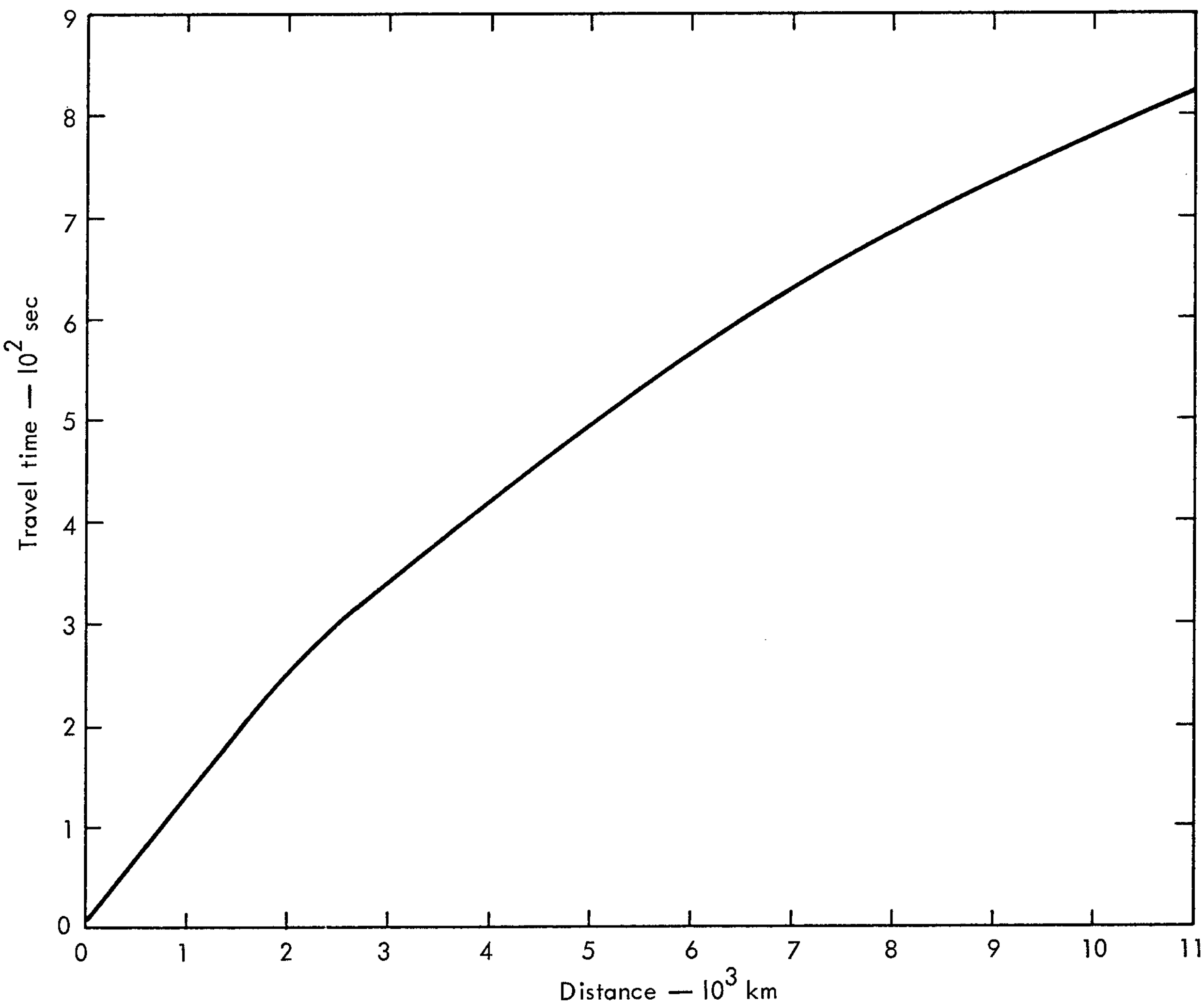

Fig. 1. Jeffreys-Bullen travel times versus distance.

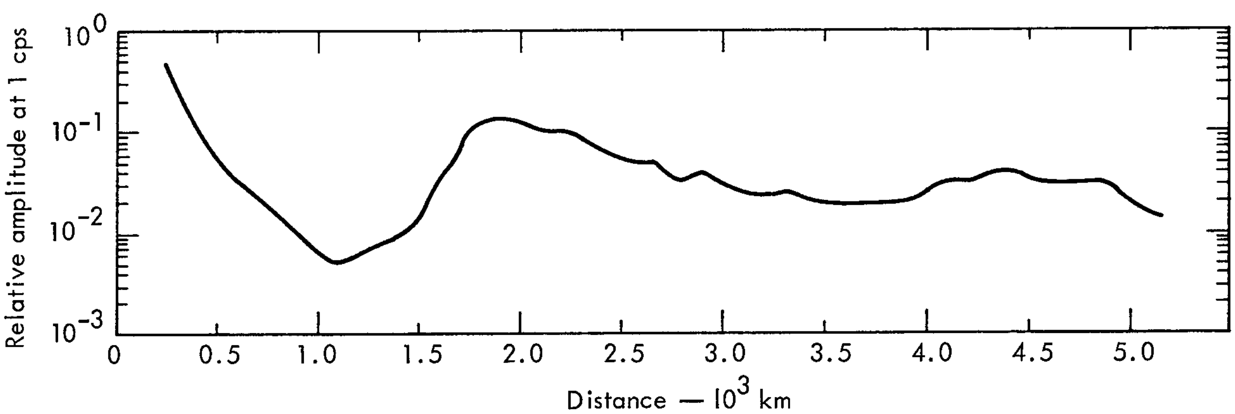

Fig. 2. AFTAC-Gutenberg curve for the relative amplitude at 1 cps for P waves versus distance.

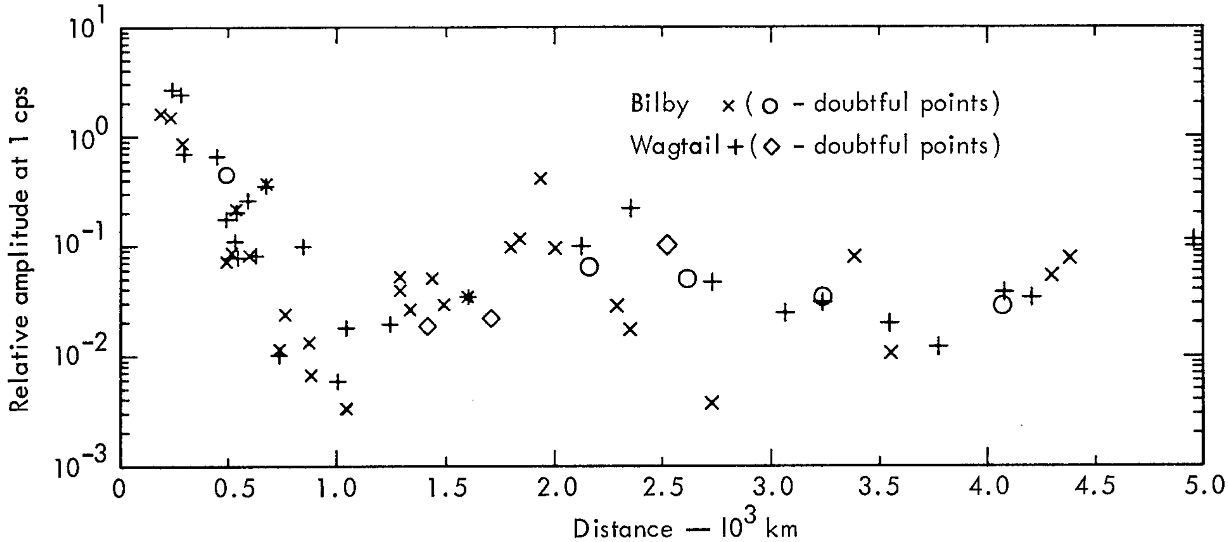

Fig. 3. Relative amplitude at 1 cps for Bilby and Wagtail tests. The Wagtail points were adjusted relative to the Bilby to take into account the different magnitudes of the two tests.

from the surface, resulting in a "shadow zone" of small amplitudes. Jeffreys[5] has criticized this model as not yielding theoretical travel times in sufficient agreement with observed values.

More recently, Werth, Herbst, and Springer[7,8] have calculated travel times and amplitudes of first arrivals out to about 750 km on the assumption that they were head waves generated by body waves critically refracted along the top of the mantle. They obtained very good agreement with experimental values of both travel times and amplitudes.

For head waves, the travel time increases linearly with distance while the amplitude diminishes monotonically. Consequently, to explain the observed change in slope of the travel time curve and the increase in the amplitudes around 2000 km, an additional mechanism must be called upon. In the present work a model was constructed upon the premise that these features could be explained by the overtaking of the head wave by a body wave that had traveled more deeply into the mantle.

For the body wave to return to the surface before the arrival of the head wave near 2000 km, the wave velocity must increase with depth below the top of the mantle. However, for the head wave to arrive first at smaller distances, there must be a region below the top of the mantle through which the velocity does not increase. The velocity-depth relationship used is shown in Fig. 4. The velocities above the mantle correspond to those of D. L. Springer's North Tatum A crustal model.[9] In the region of the mantle where the velocity increases with the depth, the velocity was assumed to follow Bullen's Law,[10,11]

$$V = V_0\left(\frac{R}{R_0}\right)^{\zeta} , \qquad (1)$$

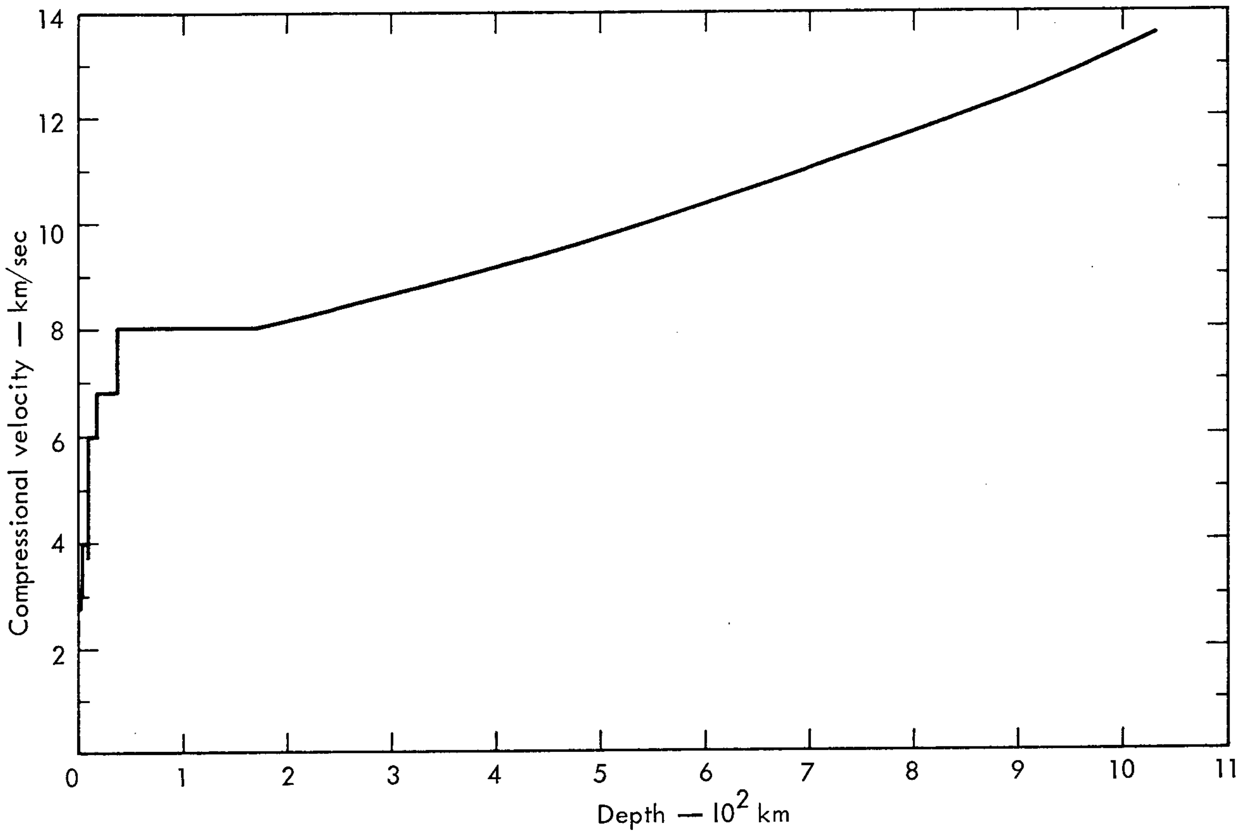

Fig. 4. Velocity-depth model.

where R and R_0 are distances from the center of the earth, and V_0 and R_0 are the values at the top of the region. Bullen's Law, with various values of ξ, can be made to approximate the actual velocity distribution over limited ranges of depth in the earth.[11] In this work, Bullen's Law was assumed to hold to a depth of 1031 km, which was the depth reached by the ray arriving at the surface at a source-detector distance of 2921 km. The calculated travel times for rays that penetrated to greater depths (and concomitantly surfaced at greater distances) deviated excessively from the experimental values.

Bullen's Law leads to simple expressions for the angular great circle distance, Δ, traversed by a body-wave ray, and the travel time, T,

$$\Delta = \frac{2}{1 - \xi} e_0, \tag{2}$$

$$T = \frac{2}{1 - \xi} \frac{R_0}{V_0} \sin e_0, \tag{3}$$

where e_0 is the angle of emergence (the complement of the angle of incidence). To these values of Δ and T were added the horizontal distance and travel time values for traversing the crust and the constant velocity zone of the mantle. The value of e_0 was obtained by estimating the change of the slope of the travel time curve at approximately 2000 to 2200 km (see Fig. 1) in the vicinity of the point where the body waves are assumed

to overtake the head waves. There was then enough information to set up a quadratic equation in ξ and in Z (Z being the thickness of the constant velocity zone of the mantle). The result that seemed to agree best with the observed values was obtained from a model based upon an estimate of 33.67° for e_0, which led to values of ξ = -3.53 and Z = 132 km for an overtake point at 1796 km.

The travel times obtained from this model are compared with the Jeffreys-Bullen times in Table I. The calculated amplitudes are compared against the AFTAC-Gutenberg curve and the Bilby-Wagtail points in Figs. 5 and 6.

Table 1. Comparison of calculated travel times with Jeffreys-Bullen times.*

Type of wave	Source detector distance, km	Calculated time, sec	J-B time, sec	Calculated time minus J-B time, sec
Head	140	25.9	24.8	1.1
"	200	33.4	32.5	0.9
"	400	58.4	58.2	0.2
"	600	83.4	83.8	-0.4
"	800	108.4	109.1	-0.7
"	1000	133.4	134.2	-0.8
"	1200	158.4	159.0	-0.6
"	1400	183.4	183.3	0.1
"	1600	208.4	207.1	1.3
Overtake point	1796	233.1	230.0	3.1
Body	1997	254.1	252.7	1.4
"	2190	273.5	273.5	0.0
"	2407	293.8	293.9	-0.1
"	2599	310.3	311.0	-0.7
"	2799	326.0	328.4	-2.4
"	2921	334.8	338.7	-3.9

*The times in this table were interpolated from the travel times given by H. Jeffreys and K. E. Bullen in Seismological Tables, British Association for the Advancement of Science, London, 1958.

The relative vertical positions of the head-wave curve and the body-wave curve were determined by calculating the appropriate geometric factors and source functions. The source function for head waves is the reduced displacement potential, $f(\tau)$, whereas the source function for body waves is

$$-\mathrm{grad}\, f = \frac{1}{V}\frac{df}{d\tau}\,, \tag{4}$$

where $\tau = t - \frac{r}{V}$, V is the compressional wave velocity in the source medium, and r is the distance from the source at which f is determined.

The relative positions of the head-wave and body-wave portions of the calculated curves do depend upon the particular reduced displacement potential function involved. In this work the two Rainier source functions of Werth, Herbst, and Springer,[7] scaled

to 5 kilotons, were convolved with the Benioff seismometer impulse response given by them to determine the relative amplitudes plotted in Figs. 5 and 6. Both their type 1 and type 2 source functions gave the same ratio of amplitudes for the first half-cycles of the body waves and the head waves: 3.69 to 1.

The source strength for body waves relative to head waves varies with the energy, W, released by the source. The reduced displacement potential has been found to vary approximately as*

$$f_a(\tau) = \frac{W_a}{W_b} f_b(\tau'), \tag{5}$$

where

$$\tau' = \left(\frac{W_b}{W_a}\right)^{1/3} \tau .$$

Then by Eqs. (4) and (5) the source function for body waves in a given medium should vary as

$$\frac{df_a(\tau)}{d\tau} = \left(\frac{W_a}{W_b}\right)\left(\frac{df_b(\tau')}{d\tau'}\right)\left(\frac{d\tau'}{d\tau}\right) = \left(\frac{W_a}{W_b}\right)^{2/3} \frac{df_b(\tau')}{d\tau'} , \tag{6}$$

or

$$\left(\frac{df_a(\tau)}{d\tau}\right)_{\tau=\tau} = \left(\frac{W_a}{W_b}\right)^{2/3} \left(\frac{df_b(\tau)}{d\tau}\right)_{\tau=\tau'} , \tag{7}$$

where the subscripts $\tau = \tau$ and $\tau = \tau'$ indicate the values of τ at which the derivatives are to be evaluated. Consequently, the ratio of body wave to head wave source strengths should vary as

$$\frac{\left(\frac{df_a(\tau)}{d\tau}\right)_{\tau=\tau}}{f_a(\tau)} = \left(\frac{W_b}{W_a}\right)^{1/3} \frac{\left(\frac{df_b(\tau)}{d\tau}\right)_{\tau=\tau'}}{f_b(\tau')} , \tag{8}$$

which means that for larger sources the body wave amplitudes should be relatively smaller in comparison with the head wave amplitudes.

*Note that there is an error in this equation in Ref. 7.

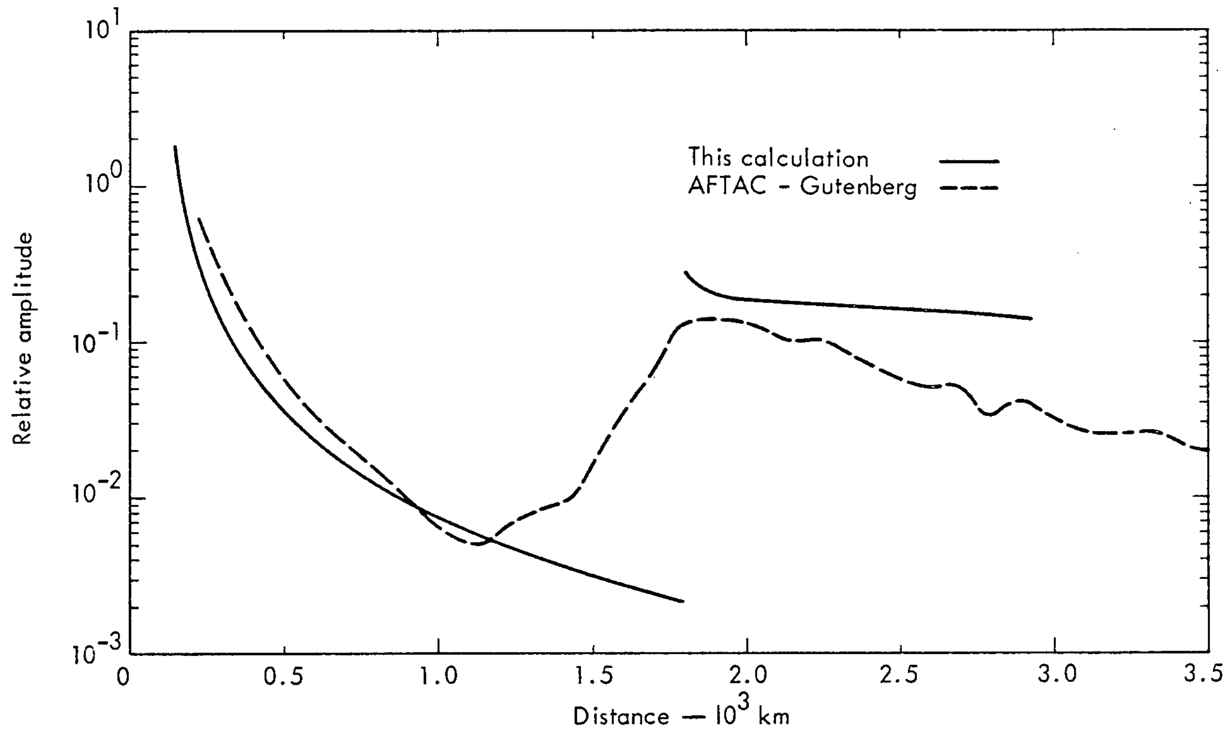

Fig. 5. Calculated relative amplitude of first half cycle of first arrival compared with AFTAC-Gutenberg curve for relative amplitude at 1 cps.

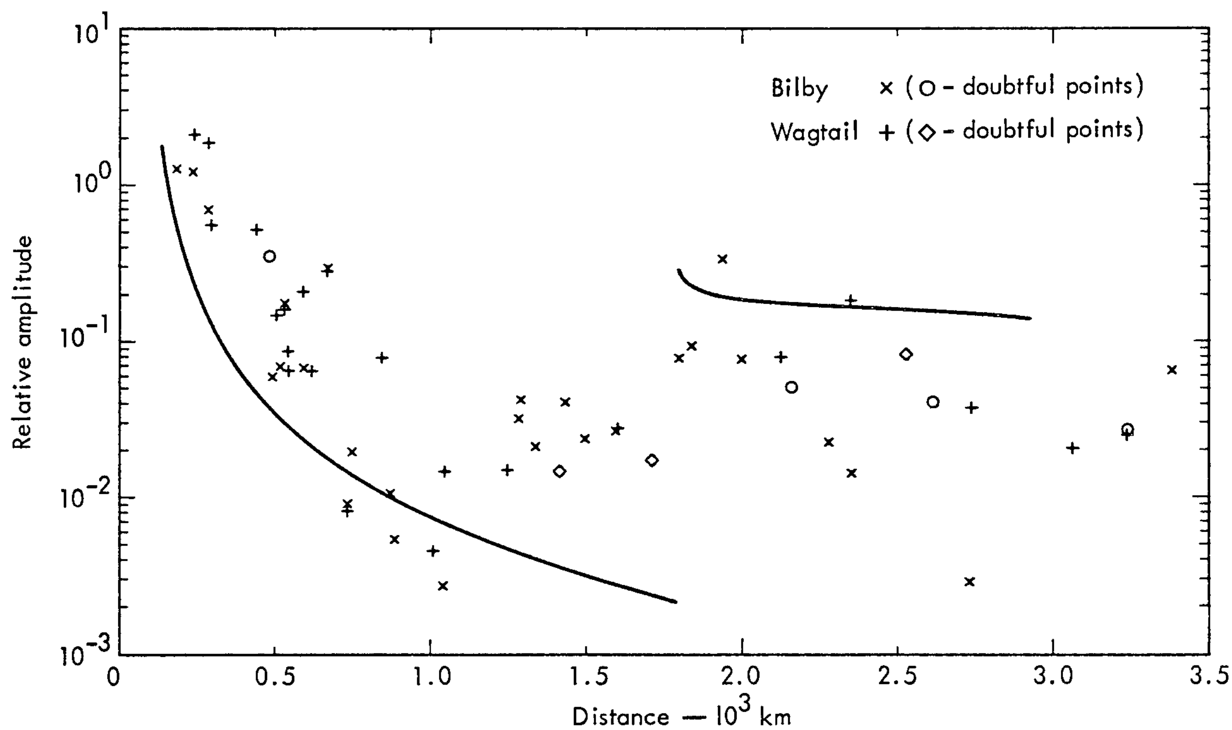

Fig. 6. Calculated relative amplitude of first half cycle of first arrival compared with relative amplitude at 1 cps recorded for Bilby and Wagtail tests. (Wagtail adjusted to Bilby.)

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[1]For the benefit of uninitiated readers, P waves are the first signals to arrive from a seismic disturbance. Other perhaps unfamiliar terminology used in this report is explained in the various references cited or in C. F. Richter, Elementary Seismology, W. H. Freeman, Co., San Francisco, 1958 or in L. M. Brekhovskikh, Waves in Layered Media, Academic Press, New York, 1960.

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